Red Feather Solar Furnace

Individual Analytical Analysis

Nathan Fisher

Fall 2020

Mechanical Engineering

Project Sponsor: Red Feather Development Group
Faculty Advisor: Dr. Oman
Sponsor Mentor: Chuck Vallance

Introduction

In order to fully understand all aspects of the solar furnace design, it is necessary to determine the total amount of heat loss out of all surfaces during the heating period of the device. Previously, the heat output for the top three design options for the casing were calculated. This previous analysis, however, did not consider all surfaces and was not calculated using the maximum temperature difference between the air in the device and the ambient air. This report contains all necessary heat loss calculations using measured values from the partially constructed device.

Previous Analysis

In the previous analysis, the top three furnace casing designs, as evaluated in the decision matrix, were compared with each other on the basis of heat loss. Each one had a slightly different geometry, so it was necessary to at least have a general idea about the insulating abilities of each one. The three geometries were, simple wooden casing, a steel and wood hybrid casing, and a wood casing with an air gap. An image of each of the three respective designs can be seen in figure A.1 of appendix A.

The scoring of the decision matrix did not consider actual calculations, but rather a general understanding of the thermal conductivity of the materials to be used. Thus, it was necessary to apply two-dimensional computations of the heat transfer out of the sides of the device for each of the three geometries above. The heat loss out of the back of the device would also be calculated however, all three systems have the same aluminum and wood design on the back so the value for this would be the same across all three designs. Technically, the heat loss will change depending on the current temperature of the device, but in order to determine the best design it is only necessary to compare the heat loss at a single temperature difference. An arbitrary value was used, with a mild expectation of the temperature in the device being 50°C above ambient.

There are a few equations that go into calculating the heat transfer out of a system. The heat transfer, q is defined by equation 1 below. In that equation, ΔT is the temperature system across the boundaries, and $\sum R_{Th}$ is the sum of the thermal resistances. There are two types of thermal resistances, one for conductive heat transfer (equation 2) and one for convective heat transfer (equation 3). In conductive heat transfer, L is the length/thickness of the wall, k is the thermal conductivity, and A is the cross-sectional area of the wall. In convective heat transfer, h is the heat transfer coefficient, and A is the cross-sectional area of the wall. These same equations will be used for the new analysis of the entire system.

$q = \frac{\Delta T}{\sum R_{Th}}$ Equation 1 [1]

$R_{conduction} = \frac{L}{kA}$ Equation 2 [1]

$R_{convection} = \frac{1}{hA}$ Equation 3 [1]

The thermal conductivity value used for wood in this instance was balsa wood with a value of 0.048 W/mK [2].The thermal conductivity of steel is 54 W/mK [3] and the thermal conductivity of air is approximately 0.025 W/mK [2]. The new computations will use the thermal conductivity of pine wood and aluminum, as that is what is being used in the actual construction of the device.

These values will be defined later. The calculations for the three casing designs can be seen in figure A.2, A.3, and A.4 of the appendix. These calculations include a dimensioned schematic, a simple resistive network, and use of the equations above. The calculated heat loss values are displayed in table 1 below.

Table 1: Heat loss of the three casing options.

Casing Design	Heat loss (Watts)
Simple wooden casing	34.01
Wood and steel hybrid	33.99
Wooden casing with an air gap	27.95

As observed in the figures above, the casing design with the lowest heat loss is the wood casing with the air gap. This result was expected, and is the reason that design scored the highest on the decision matrix (appendix A). The air gap design results in a 27.95 Watt heat loss at the chosen temperature. Both of the other designs result in approximately 34 Watts of heat loss. This is a obvious improvement; however the higher efficiency needs to be weighed against the complexity of adding the air gap. These calculations were taken into consideration and compared against the added manufacturing cost and difficulty, and it was decided that the simple wooden casing was the best option since 6 Watts was not an immense loss in comparison to the 1500 Watt output of the device.

Further Calculations & Results

As stated above, now that the design components have been finalized it is necessary to compute the heat loss out of all surfaces of the solar furnace. This value is important to ensuring the device produces the desired heat output. Since many of the materials have been acquired there are new thermal conductivity values that will need to be used in the computations. These values can be seen below in table 2. The coefficients of convection are 10 W/m^2K and 50W/m^2K for the ambient air and the air in the furnace respectively [1].

Table 2: Thermal Conductivity values [2,3].

Material	Thermal Conductivity (W/mK)
Pine Wood	0.12
Aluminum	164
Acrylic	0.20

One other major change to the calculations is that they will be computed based off the maximum expected temperature difference between the air in the device and the ambient air. This value is expected to be approximately 75°C. Also, the inner dimensions are 33inches by 69inches, with the outer dimensions being 36inches by 72inches. All necessary calculations, utilizing the equations introduced above, were computed in MATLAB. The published live script, including all equations, calculations, schematics, and resistive networks, can be found in Appendix B at the end of this report. With MATLAB's computational functionality, it was found that the total theoretical heat loss of the entire system is approximately 3.7kW. The loss out of each surface, and the summation of those losses, can be found individually within the published live script.

Conclusion

Obviously this number is concerning, seeing as the output of the system is only 1.5 kW, less than 1 half of the total losses. This is a theoretical calculation and should be taken at face value but nonetheless is an important factor to consider. Some assumptions that could be made to reduce this value would be that the backing of the device is butted up against the home, so the losses there may be much lower than predicted. Also, the air within the device is at a standstill until the fan is activated, and the calculations performed were considering forced convection. The majority of heat loss is out of the acrylic panel on the front so the design team will explore the possibility of adding a secondary layer with an air gap in between. One other option is that of added insulation to the back. Lastly this value is that of the max losses on the coldest day of the year in Page, AZ. The device still needs to perform well on these days, but it will not be constantly losing heat at this maximum rate. The fan speed can be increased to reduce the heat loss due to forced convection when the device is at maximum temperature. It is also important to consider that the device has been oversized for the necessary heat output to begin with. These calculations will be reviewed, and if necessary corrected, with the design team and will be integrated into the continued construction of the device to reduce these losses. The team will also look into the necessity of a full-scale nodal analysis of the system to more accurately describe the heat loss.

References

[1] T. L. Bergman, A. Lavine, and F. P. Incropera, *Fundamentals of heat and mass transfer 8th ed*. Hoboken, NJ: John Wiley & Sons, Inc., 2019.

[2] Thermal Conductivity of Selected Materials and Gases. Engineeringtoolbox.com, 2020. [Online] Available: https://www.engineeringtoolbox.com/thermal-conductivity-d_429.html [Accessed 8 Oct 2020].

[3] Thermal Conductivity of Metals, Metallic Elements and Alloys. Engineeringtoolbox.com, 2020. [Online] Available: https://www.engineeringtoolbox.com/thermal-conductivity-metals-d_858.html [Accessed 8 Oct 2020].

Appendix A: Previous Analysis

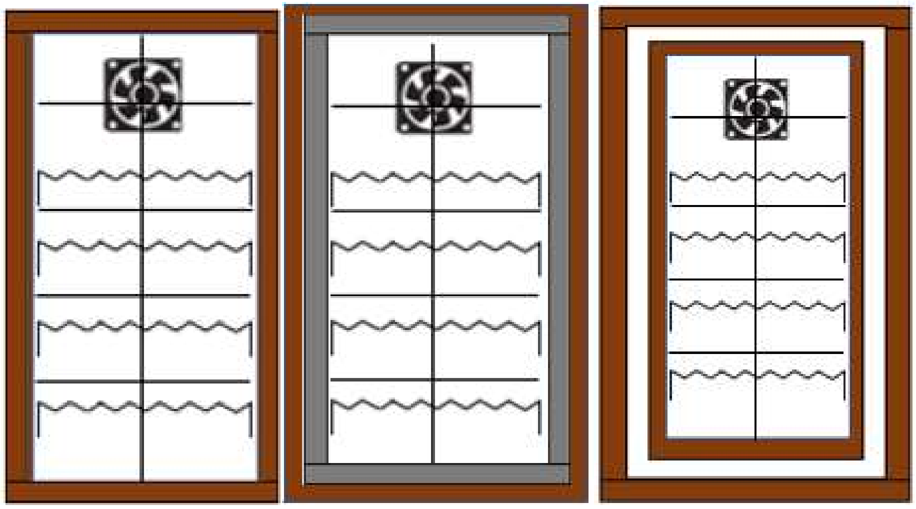


Figure A.1: Top three solar furnace casing options.

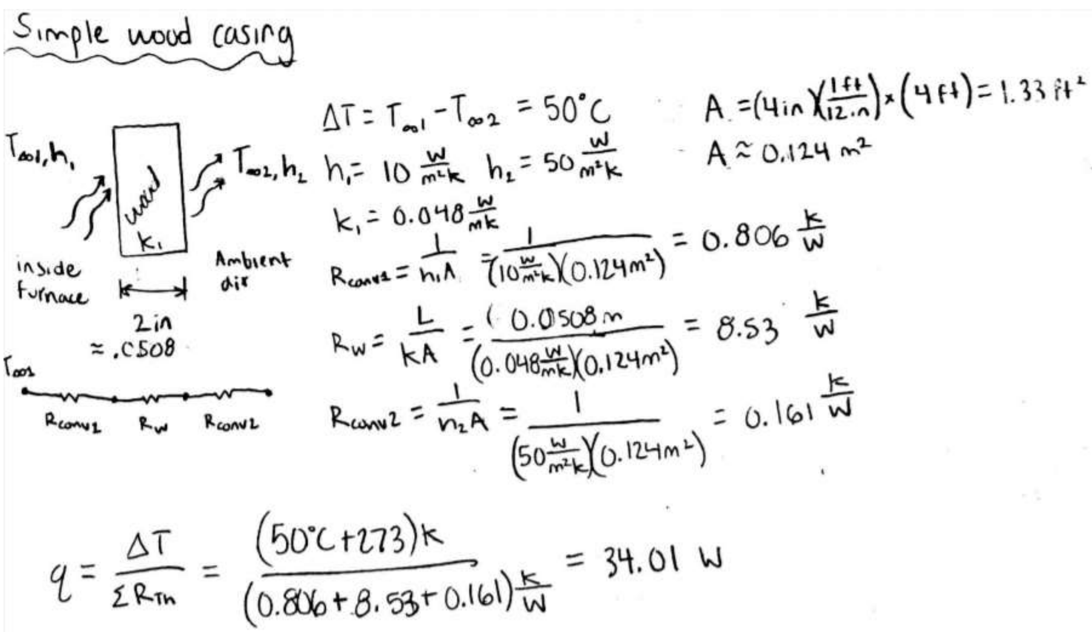


Figure A.2: heat loss calculation for simple wooden casing.

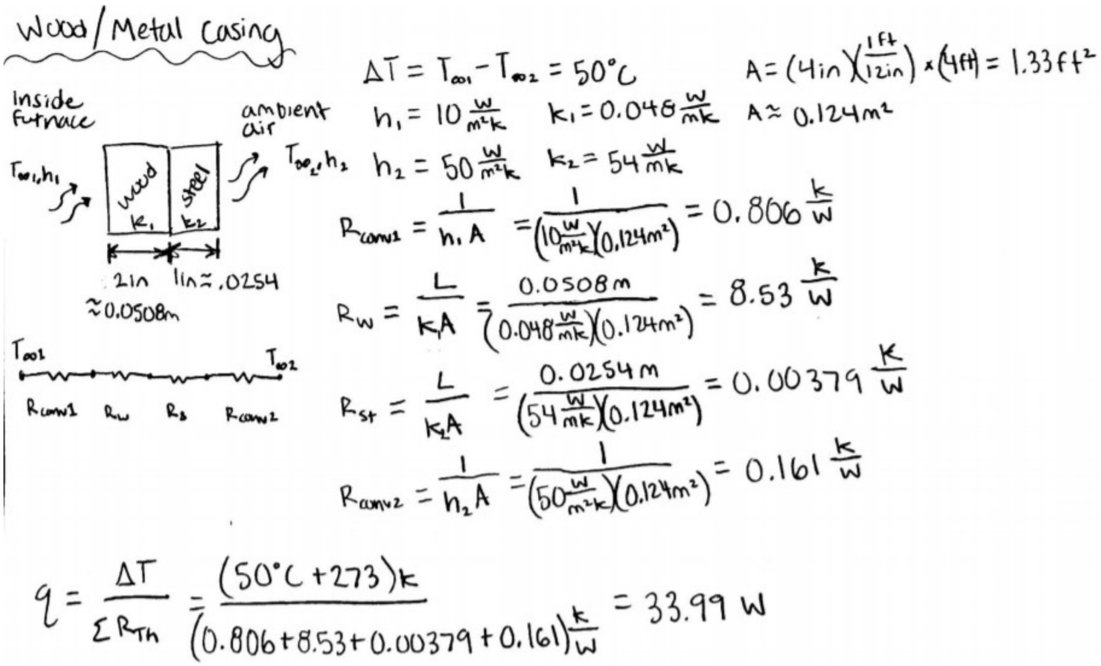


Figure A.3: heat loss calculation for wooden casing with metal inner layer.

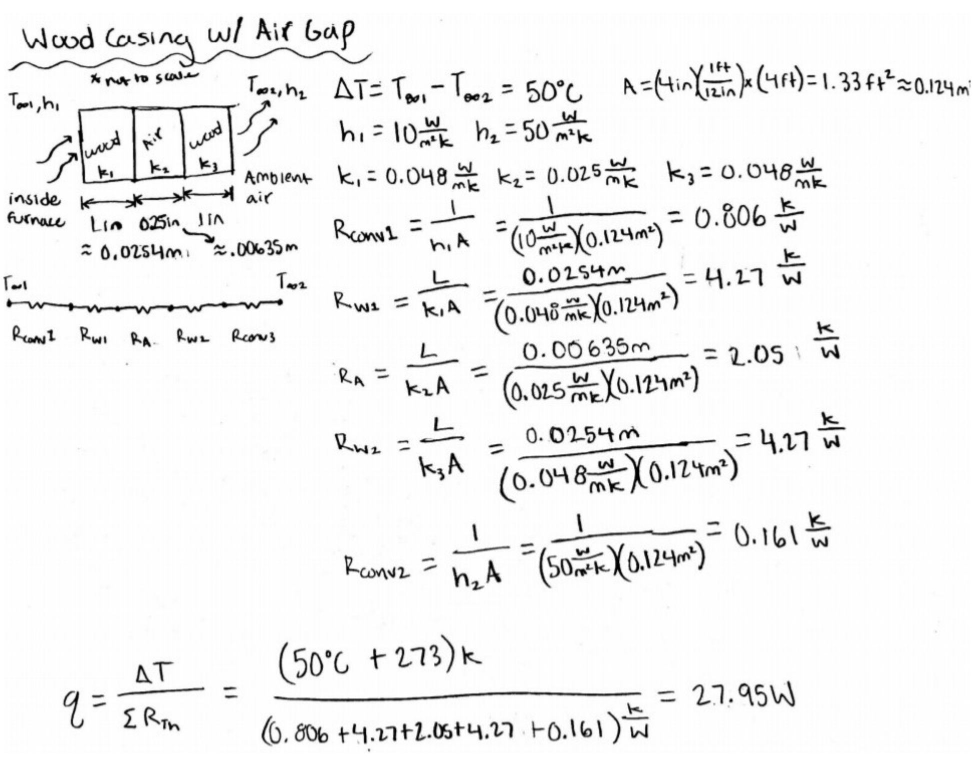


Figure A.4: heat loss calculation for wooden casing with air gap.

Appendix B: Device Schematics & MATLAB Calculations

System Heat Loss Calculations

Author: Nathan Fisher

Givens/Constants

```
k_wood = 0.12; %W/mK
k_Al   = 164;  %W/mK
k_Acr  = 0.20; %W/mK
h1 = 10;       %W/m^2K
h2 = 50;       %W/m^2K
```

Necessary Equations

Assume steady state

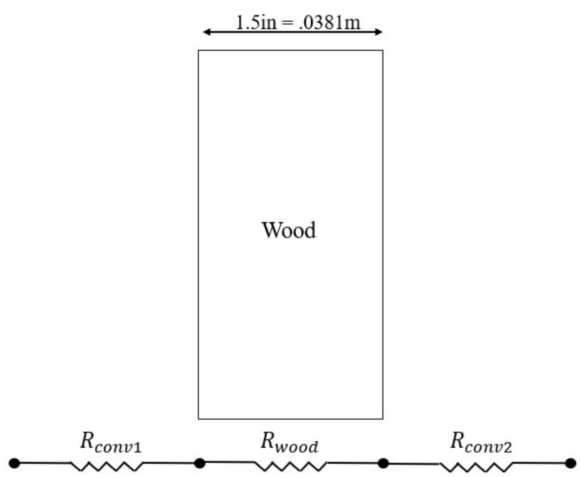
$$q = \frac{\Delta T}{\Sigma R_{\text{th}}}$$

$$R_{\text{conduction}} = \frac{L}{\text{kA}}$$

$$R_{\text{convection}} = \frac{1}{\text{hA}}$$

```
delta_T = 75 + 273; %Kelvin
```

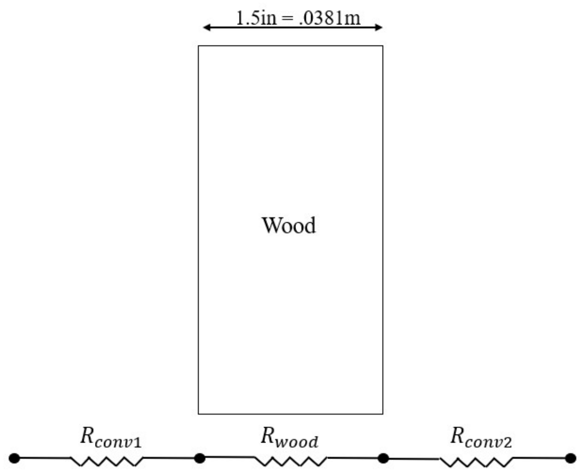
Siding: Long (quantity 2)




```
A1 = .0889 * 1.7526;                %m^2
A2 = .0889 * 1.8288;                %m^2
R_conv1 = 1 / (h1 * A1);            %k/W
R_wood = 0.0381 / (k_wood * A1);    %k/W
```

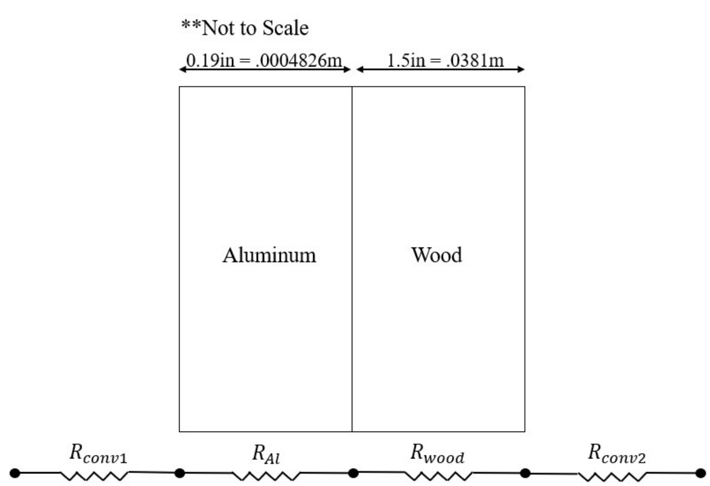
```
R_conv2 = 1 / (h2 * A2);            %k/W
R_sum = R_conv1 + R_conv2 + R_wood; %k/W
q1 = delta_T / R_sum                %W
```
```
q1 = 124.1692
```

Siding: Short (quantity 2)




```
A1 = .0889 * .8382;                 %m^2
A2 = .0889 * .9144;                 %m^2
R_conv1 = 1 / (h1 * A1);            %k/W
R_wood = 0.0381 / (k_wood * A1);    %k/W
R_conv2 = 1 / (h2 * A2);            %k/W
R_sum = R_conv1 + R_conv2 + R_wood; %k/W
q2 = delta_T / R_sum                %W
```
```
q2 = 59.4988
```

Back Panel

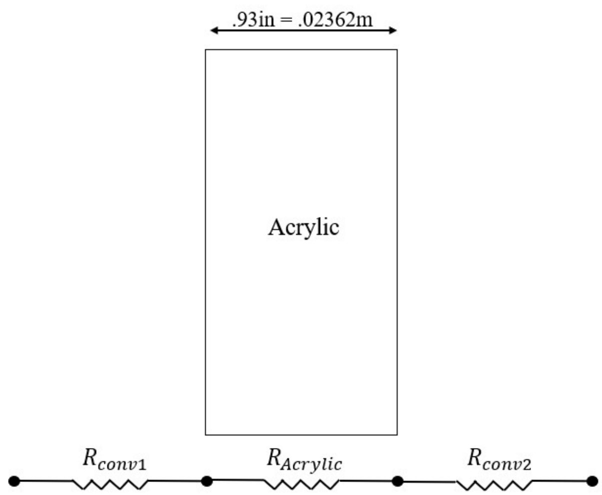



```
A = 1.7526 * .8382;                        %m^2
```

```
R_conv1 = 1 / (h1 * A);                      %k/W
R_Al = 0.0004826 / (k_wood * A);             %k/W
R_wood = 0.0381 / (k_wood * A);              %k/W
R_conv2 = 1 / (h2 * A);                      %k/W
R_sum = R_conv1 + R_conv2 + R_Al + R_wood;   %k/W
q3 = delta_T / R_sum                         %W
```

q3 = 1.1579e+03

Front Panel




```
A1 = 1.7526 * .8382;                 %m^2
A2 = 1.8288 * .9144;                 %m^2
R_conv1 = 1 / (h1 * A1);             %k/W
R_Acr = .02362 / (k_Acr * A1);       %k/W
R_conv2 = 1 / (h2 * A2);             %k/W
R_sum = R_conv1 + R_conv2 + R_Acr;   %k/W
q4 = delta_T / R_sum                 %W
```

q4 = 2.1692e+03

Total Heat Loss

```
q_sum = (2 * q1) + (2 * q2) + q3 + q4 %W
```

q_sum = 3.6944e+03